

Does Path Induction Need a Justification?

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In their papers, Ladyman and Presnell [2015] and Walsh [2017] argue that path induction, the definition of identity in Homotopy Type Theory (HoTT), needs a philosophical justification. The former claims that, since HoTT is an autonomous foundation for mathematics, path induction must be justified from a pre-mathematical notion of identity. The latter claims that path induction must be justified by the notion of harmony between the introduction and elimination rules of identity types. I argue against these two positions and claim that path induction does not need a further philosophical justification.

I argue that we ought to understand path induction from the perspective of homotopy type theorists, in particular the authors of the book *Homotopy Type Theory* (Univalent Foundations Program [2013]). According to the Univalent Foundations Program [2013], HoTT is understood from the *intrinsic homotopical content* of types [p.5]. I prove that path induction follows from the path lifting property from the homotopical perspective, and thus it does not require a further philosophical justification as claimed by Ladyman and Presnell [2015] and Walsh [2017].

References

James Ladyman and Stuart Presnell. Identity in Homotopy Type Theory, Part I: The Justification of Path Induction. *Philosophia Mathematica. Series III*, 23(3):386–406, 2015.

Patrick Walsh. Categorical Harmony and Path Induction. *Review of Symbolic Logic*, 10(2):301–321, 2017.

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