UNIVALENT FOUNDATIONS AND THE CONSTRUCTIVE VIEW OF THEORIES

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Univalent Foundation (UF) has been recently proposed as a novel foundation of mathematics. We explore a possibility of using UF beyond the pure mathematics as a general formal semantic framework for representing scientific theories. This project is parallel to the project of formal semantic representation of theories by means of Bourbaki-style set-theoretic foundations of mathematics and Tarski-style Model theory, which has been started by Suppes back in 1950-ies and is presently known under the name of "semantic view of theories" [5], [1]. We argue that UF as a prospective representational tool for science and technology has important advantages since it allows for a uniform mathematical representation of various extra-logical methods, which are abound in these fields. This leads us to a new view of theories that at the absence of a better name we call constructive [3]. According to this view a scientific theory is essentially characterised by its *methods* including the methods of verification and justification of its theoretical statements. Interestingly, a similar view of theories was earlier defended by Ernest Nagel in 1930-ies [2] even if at that time there were no available means for expressing and implementing this view at the formal level.

The Axiomatic Method as construed by David Hilbert in the very beginning of the 20th century and further developed during the same century proved to be a powerful tool for a meta-theoretical mathematical study of deductive and expressive aspects of theories. At the same time its intended role as a working method of organising and representing various scientific theoretical contents in a logically transparent form, which could serve a

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working mathematicians and working scientists for formal verification and communication of new results, writing textbooks and other similar "practical" purposes, has been only poorly realised in the mathematical and scientific practices of the same century. The early 20th century enthusiasm about prospects of Axiomatic Method in mathematics and sciences in the second half of this century was followed by a widespread scepticism about the significance of logical methods in these fields. This applies, in particular, to the "semantic" version of the Axiomatic Method developed by Suppes and his followers. The recent experience with UF, which involves a workable representation of a significant body of todays mathematics with a computer code (see, in particular, the UniMath library), demonstrates good prospects for UF as a "practical" foundation of mathematics. We identify some conceptual reasons of this success and argue that the same reasons support a possibility of using UF as a representational framework beyond mathematics.

A detailed argument and discussion are found in [4]

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