

Synthetic Homology in Homotopy Type Theory

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This talk will sketch some original research defining homology in homotopy type theory. Our approach is similar to Evan Cavallo's definition of cohomology. For each type X , we want a group $H_n(X)$ that satisfies the Eilenberg–Steenrod axioms of homology. To this end we would like a definition of H_n that does not depend on the cell structure of a space. Luckily there is such a construction in the literature and we will broadly follow the classical approach. However, since our definitions apply to types, our proofs will often be unique, combining ideas from type theory with ideas from classical homotopy theory. This talk will highlight the differences from the classical theory.

To be precise, we will be showing the following new results in synthetic homotopy theory. To begin with, we define stable homotopy groups. To our knowledge stable homotopy theory has not previously been defined in the literature, but the main tools for working with them have already been fleshed out (in particular the Freudenthal suspension theorem and the Blakers-Massey theorem). Our main result is proving the stable homotopy groups $\pi_n^s(-)$ form a homology theory. After which we can show $\operatorname{colim}_i \pi_{n+i}^s(X \wedge K_i)$ is a homology theory for a fixed spectrum K_i . Regular homology $H_n(X, G)$ is then defined using the spectrum of Eilenberg-MacLane spaces $K(G, i)$. This work has not been implemented in a proof checker but we will mention progress in that direction made by other researchers.